# Application of Polyspectral Techniques to Nonlinear Modeling and Compensation

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*Abstract* — **This paper introduces polyspectral techniques to the microwave community by describing their application to nonlinear component and system modeling, as well as compensation design. Basic features of the method are provided, and a specific application is given to the black-box modeling of a commercial 20-GHz traveling-wave tube amplifier using 9.6 Gbps, 16-APK input signals.**

## I. Introduction

In both commercial and military sectors, the demands for information services have escalated rapidly, requiring ever more efficient use of power and limited channel bandwidths. With respect to the power issue, by far the most attention has been paid to the enhancement of the power efficiency in transmitter and transponder *high-power amplifiers* (HPAs). In both *solid-state power amplifiers* (SSPAs) and *traveling-wave tube amplifiers* (TWTAs), this goal is primarily and most easily achieved by operating the HPA nonlinearly. Unfortunately, this also leads to unwanted distortion in the modulated signal passing through the HPA that in turn must be compensated in some way. This distortion is further exacerbated by multi-level signals that are required for desired bandwidth efficiency and multi-user services, as well as the wider bandwidths needed to accommodate information throughputs. For SSPAs, a variety of approaches have been proposed to address this challenging tradeoff between high efficiency and low distortion [1]. For TWTAs, a higher-level system approach is normally used to compensate for distortion, as exemplified in [2].

In order to systematically address the design and performance evaluation of these advanced communication systems, efficient and accurate nonlinear component and channel simulation models must be developed (see [3], for example). These tools are also essential to the development of effective compensators since they necessarily attempt to invert the response of a component or channel segment. The focus in this endeavor has been on HPA characterization, driven by the explosive growth in personal communication systems that are subject to the distortion sensitivities just discussed. There are three basic classes of HPA models that have been proposed, namely, physics-based models, equivalent-circuit models, and block system models. All three classes appear for SSPAs, while mainly the first and third class is found for TWTAs. The accuracy of the models decreases from the first class to the last, with a reverse trend found for the simulation times to run the model, and the amount of measurements needed in its construction. These characteristics, coupled with the need for system-level performance trade studies which require many simulation runs, have caused most attention to be paid to block system models. In addition, most satellite communication systems include TWTAs that are really only amenable to block system models.

To date, the majority of nonlinear HPA block models are constructed from sinusoidal measurements and simple variations thereof [3]. This simplicity allows for the use of *vector network analyzers* (VNAs) which can make very accurate measurements at high frequencies. Recently, the development of a likewise accurate *baseband time-domain measurement technique* (BTDMT) [4, 5] makes possible the evaluation of these models, as well as the development of new time-domain models. This paper will build on the previous announcement in [6] by presenting a powerful time-domain system identification technique suitable for both modeling and compensation design.

## II. The Polyspectral Approach

The polyspectral approach developed in the spectral analysis community with its primary application area being the identification of linear and nonlinear mechanical systems [7, 8]. In essence, the approach employs time-domain input/output measurements and spectral calculations to arrive at a formal characterization that treats the given system as an input/output functional operator (with memory). For linear systems, this results in the familiar and global impulse response. In the nonlinear case, the characterization is only locally applicable and in the form of a series for the operator in a function-space setting. In fact, the basic forms of the polyspectral model can be shown to be special single-frequency cases of the general Volterra description that locally generalizes the linear impulse response for nonlinear systems [9].

The polyspectral model architecture consists of two or more parallel signal paths from the input to the output, one path of which is linear while the remaining paths are nonlinear. The nonlinear paths consist of a two-box structure made up of a chosen

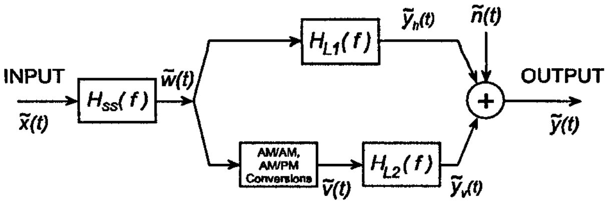


Figure 1: Variant nonlinearity-filter structure used for HPA modeling.

memoryless nonlinearity together with a linear filter, the order of which determines whether the model is of the *nonlinearity-filter* (NF) or *filter-nonlinearity* (FN) type. A hybrid version of the model consisting of both types of nonlinear paths can also be proposed. In their original usage context, these models apply to real signals and have been modified here to accommodate complex baseband signals. A modified version of the basic NF-type model is shown in Figure 1 for use in modeling TWTAs. In all cases, the filters serve as the unknown model parameters to be determined from the input/output measurements. The chosen nonlinearities are limited to simple monomials for the FN type, while the NF type allows for general memoryless nonlinearities (such as the traditional AM/AM and AM/PM envelope conversion curves standardly used for HPAs).

There are several important features and advantages of this approach that make it natural for nonlinear modeling and compensation design. These observations were partially covered in [6] for the first optimal-filter step of the polyspectral approach, while [7] provides a fully detailed treatment. First, the model parameters have closed-form expressions that are globally optimal in minimizing the error power between the model predictions and measured outputs. Second, the approach provides a means for quantifying its own fidelity, and added nonlinear branches are rigorously guaranteed to increase this fidelity through a decorrelation procedure. Third, complete statistical bias error estimates (systematic and random) are provided for the model parameters that aid in the design of experiment for the construction measurements (such as the number of input/output records $N_r$ to measure). These measurements employ random-like stimulus signals with well-defined properties, which for the simpler NF type can simply consist of pseudo-randomly modulated signals. Finally, the method also provides for the straightforward development of an analytical link model useful for bit-error rate bounding and nonlinear equalizer design. In a similar way, the method is very well suited for compensation design since the inverse model is easily obtained with a structure that is natural for hardware implementation.

## III. Nonlinearity-Filter HPA Model Example

To provide a concrete example of the polyspectral approach, a model was developed that is especially effective for characterizing TWTAs (see Figure 1). This model was derived from a general instance of the NF type model that consists of two branches, with the nonlinear branch containing an arbitrary memoryless nonlinearity. The original form of the polyspectral model involves real-valued signals, which was modified to handle the complex baseband signals measured by the BTDMT cited above. These signals are denoted by a tilde (' $\tilde{}$ ') over the signal variable, and are of the form $\tilde{z}(t) = z_I(t) + jz_Q(t)$, where the real (imaginary) part is the in-phase (quadrature) component. The general nonlinearity here was taken to be the standard AM/AM and AM/PM curves measured at centerband using a VNA. Finally, from a knowledge of TWTA physics that dictates that the tube's nonlinear amplification occurs near the output, a small-signal filter $[H_{ss}(f)]$ was placed in front of the standard polyspectral architecture that represents a small-signal VNA sweep of the TWTA. The unknown lowpass equivalent model filters are represented by $H_{Li}(f),\ i = 1, 2$, while $\tilde{n}(t)$ represents the modeling error between the model's predicted output of $\tilde{y}_h(t) + \tilde{y}_v(t)$ and the measured output $\tilde{y}(t)$.

The model is constructed as follows. Besides the VNA measurements used in obtaining $H_{ss}(f)$ and the memoryless nonlinearity, $N_r$ baseband input/output baseband measurements are made with a duration of $T$, the inverse of which dictates the frequency resolution of the unknown filters. The model filters are then calculated from the closed-form expressions:

$$H_{L1}(f) = H_o(f) - C(f), \quad H_{L2}(f) = \frac{S_{\tilde{u}\tilde{y}}(f)}{S_{\tilde{u}\tilde{u}}(f)}, \qquad (1)$$

where

$$H_o(f) = \frac{S_{\tilde{w}\tilde{y}}(f)}{S_{\tilde{w}\tilde{w}}(f)}, \quad C(f) = \frac{S_{\tilde{w}\tilde{v}}(f)}{S_{\tilde{w}\tilde{w}}(f)} H_{L2}(f), \qquad (2)$$

represent the optimal filter (see [6]) between the small-signal filter output $\tilde{w}$ and $\tilde{y}$, and for the nonlinear model branch, respectively, and

$$S_{\tilde{u}\tilde{y}}(f) = S_{\tilde{v}\tilde{y}}(f) - \frac{S_{\tilde{v}\tilde{w}}(f)}{S_{\tilde{w}\tilde{w}}(f)} S_{\tilde{w}\tilde{y}}(f) \qquad (3a)$$

$$S_{\tilde{u}\tilde{u}}(f) = S_{\tilde{v}\tilde{v}}(f)[1 - \gamma^2_{\tilde{w}\tilde{v}}(f)] \qquad (3b)$$

$$\gamma^2_{\tilde{w}\tilde{v}}(f) = \frac{|S_{\tilde{w}\tilde{v}}(f)|^2}{S_{\tilde{w}\tilde{w}}(f) S_{\tilde{v}\tilde{v}}(f)} \qquad (3c)$$

Here $S_{\tilde{a}\tilde{a}}(f)$ $[S_{\tilde{a}\tilde{b}}(f)]$ is an *autospectral density* [*cross-spectral density*] smoothly estimated from the expressions:

$$S_{\tilde{a}\tilde{a}}(f) = \frac{1}{N_r T} \sum_{k=1}^{N_r} |A_k(f,T)|^2 \qquad (4a)$$

$$S_{\tilde{a}\tilde{b}}(f) = \frac{1}{N_r T} \sum_{k=1}^{N_r} A_k^*(f,T) B_k(f,T) \qquad (4b)$$

where $A_k(f,T)$ and $B_k(f,T)$ are finite Fourier transforms of the $k$th data record for $\tilde{a}(t)$ and $\tilde{b}(t)$, respectively, and are easily calculated using a standard FFT algorithm. The quantity $\gamma^2_{\tilde{w}\tilde{v}}(f)$ in (3c) is an example of a *linear coherence function* (LCF) that quantitatively measures the degree of linearity of a given nonlinearity (in this case, the memoryless nonlinearity in Figure 1; see [6]). It is important to note that formally, the two filters in (1) globally minimize the autospectral density of $\tilde{n}(t)$.

In this case, the polyspectral model fidelity is quantified by a frequency-dependent relative error metric given by

$$E_r(f) = \frac{S_{\tilde{n}\tilde{n}}(f)}{S_{\tilde{y}\tilde{y}}(f)} = 1 - \gamma^2_{\tilde{x}\tilde{y}}(f) - q^2_{\tilde{x}\tilde{y}_u}(f), \qquad (5a)$$

where

$$\gamma^2_{\tilde{x}\tilde{y}}(f) = \frac{S_{\tilde{y}_o\tilde{y}_o}(f)}{S_{\tilde{y}\tilde{y}}(f)}, \quad q^2_{\tilde{x}\tilde{y}_u}(f) = \frac{S_{\tilde{y}_u\tilde{y}_u}(f)}{S_{\tilde{y}\tilde{y}}(f)}, \qquad (5b)$$

is an LCF and a *nonlinear coherence function* (NCF), respectively, where

$$Y_o(f) = H_o(f)X(f), \; Y_u(f) = H_{L2}(f)V(f) - C(f)X(f) \qquad (5c)$$

represent uncorrelated branch outputs. Like the LCF, the NCF quantifies the contribution of the nonlinear model branch towards the total input/output relationship. In addition, a very useful time-domain waveform metric, termed the *normalized mean square error* (NMSE) [5], can also be used to assess the accuracy of a model's predictions. This metric is essentially the error power between the measured and modeled waveforms normalized by the power of the measured waveform.

## IV. Application To A Commercial TWTA

The above model was constructed and evaluated for a commercial 20-GHz TWTA operating near saturation with 9.6 Gbps, 16-*Amplitude-Phase Keying* (APK) modulated input signals. The BTDMT was used to measure 100 input/output time-domain records of length 40 ns (corresponding to a frequency resolution of 25 MHz) with a sampling interval of 26 ps. An output coupler assembly of approximately $-60$ dB attenuation and parabolic phase was added to the TWTA in order to accommodate dynamic range limitations for the measurements. The statistical variation between the input data records was obtained by using 96 bit pseudorandom data sequences. The first 95 records were used for the construction, while the remaining five were used for NMSE evaluation.

Figure 2 provides the model small-signal prefilter and memoryless nonlinearity derived from standard VNA measurements of the TWTA and output coupler. The magnitudes of the calculated linear and nonlinear branch filters are given in Figure 3. Note that the filters are about at the level of the output coupler attenuation. The model evaluation results are presented in Figure 4. Observe that the relative modeling error in part (a) of the figure is lowest in the midband region and increases towards the band edges. Part (b) of the figure compares the NMSE of the new model with

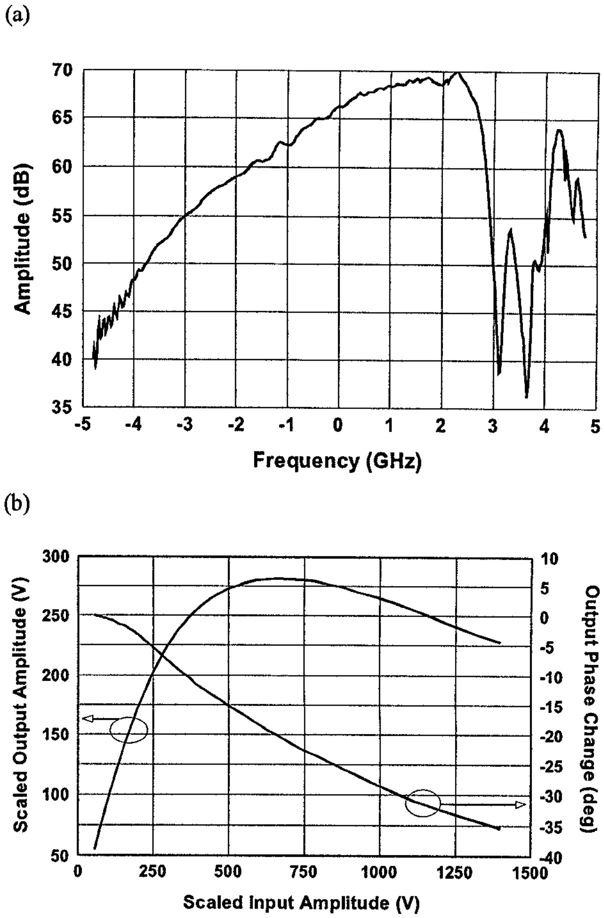


Figure 2: Basic VNA measurements of a commercial TWTA. (a) Small-signal filter magnitude response. (b) Normalized AM/AM and AM/PM conversion curves.

that of the standard two-box model for the TWTA that consists of the small-signal filter followed by the memoryless nonlinearity from Figure 2. Note that the polyspectral model in Figure 1 contains this standard two-box model. There is an improvement of over 8.5 dB average NMSE gained by the new model, where $-30$ dB NMSE corresponds to a relative error of 0.1% in the time-domain waveforms. Such high accuracies are required for BER link simulations with these complex modulations.

## V. Conclusion

This paper has introduced the polyspectral approach as a new means of HPA and link system modeling, as well as a systematic means of designing and analyzing distortion compensators. The basic elements and advantageous features of the approach were presented that strongly motivate its application to complex modulation and high-nonlinearity communications contexts. A new two-branch nonlinearity-filter variant of the model was proposed that is especially applicable to commonly-

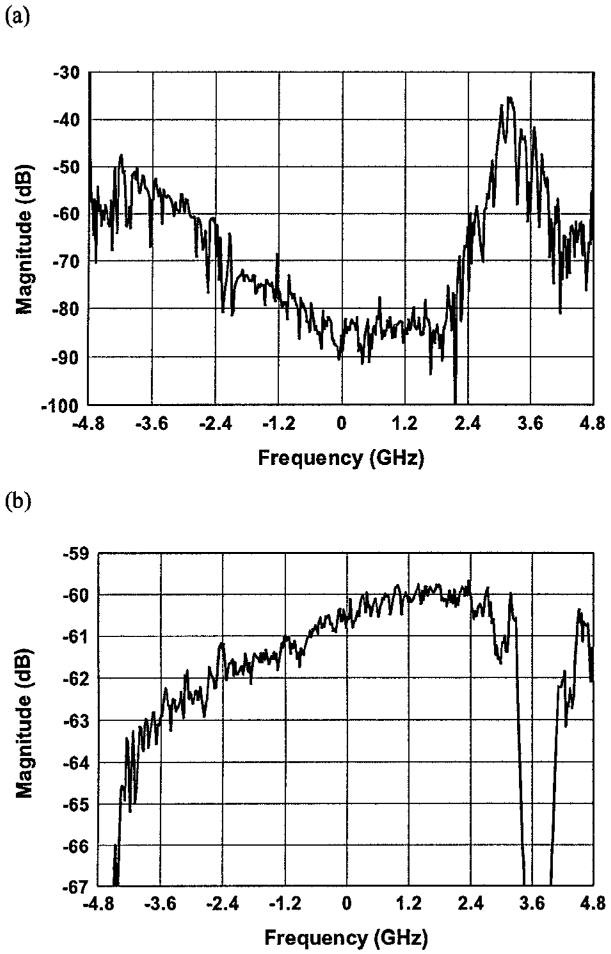


Figure 3: Calculated model component filters. (a) Linear branch filter magnitude response. (b) Nonlinear branch filter magnitude response.

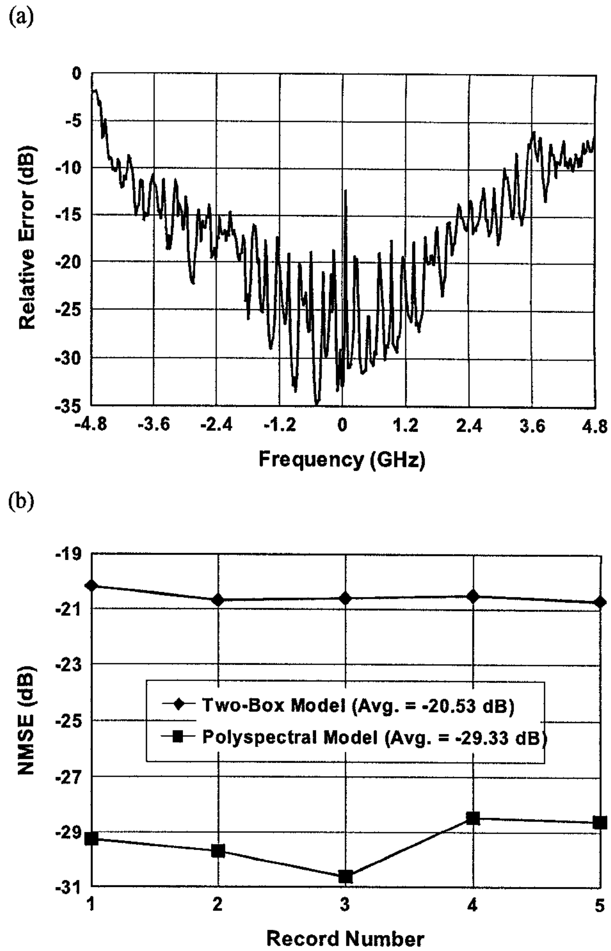


Figure 4: Model evaluation results for 9.6 GHz, 16-APK saturating signals. (a) Relative modeling error for 95 construction records. (b) NMSE comparison for remaining five records.

used TWTA HPAs. Explicit construction and evaluation details were provided and a specific application was made to a 20-GHz commercial TWTA operated in a wideband saturated mode. The new model proved significantly superior in NMSE fidelity to the standard and widely used two-box model. Further evaluation and development of this approach will lead to an important new tool in the development and assessment of broadband communication systems employing nonlinear HPAs and compensators.

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